Math Field Day 2016

Non Routine Problem Solving Level 2

Place all answers on the line provided. Give simplified EXACT answers. Calculators are permitted. You are not expected to answer all of the questions.

1. If $5$ minus the reciprocal of $5 - x$ equals the reciprocal of $x + 5$, what is $x$?

2. Two cars start at the same point on a straight highway facing opposite directions. Each drives 12 miles, takes a 90° left turn, and drives 5 miles. How far apart are the two cars?

3. Four positive integers have a sum of 78. What is the greatest possible product of these five numbers?

4. In how many zeros does $(8)^6(15)^7$ end?

5. The difference between two real numbers is 2, while the difference between their cubes is 386. Find the numbers.

6. Find how many integers between 1 and 500 are divisible by all of the numbers 4, 5, and 6.

7. What is the maximum value of the product $2xy$ if $x + 4y = 500$?

8. A regular hexagon has an area $30\sqrt{3}$ square cm. on each side of this hexagon, an equilateral triangle is constructed. What is the total area of the resulting shape?

9. Solve for $x$ the equation $\log_3(\log_5(2x + 1)) = 1$.

10. Suppose $a$ and $b$ are positive integers such that $\frac{a}{3} + \frac{b}{5} = \frac{17}{15}$.