2016 Math Field Day Trigonometry (Open)

State the exact answer for each problem. Do **NOT** use decimal approximations for π, \( \sqrt{2} \), \( \sqrt{3} \), for example. Radicals must be in simplest radical form and fractions in simplest form, or lowest terms. If there are multiple solutions, separate them by a comma. If there is no solution, write 'NONE'.

1. ____________ Given \( \tan \theta = 2 \) and \( \sin \theta < 0 \), find the exact value of \( \sec \theta \).

2. ____________ The circle with center \( O \) has radius 2 in and the angle which sector \( AOB \) subtends is 60°. Find the area of sector \( AOB \).

3. ____________ Solve \( \sin x + \cos x \cot x = -\sqrt{2} \) in the interval \([0, 2\pi)\). Write your answer in radian measure.

4. ____________ Rewrite the angle \( \left( \frac{\pi}{4} \right) \) in radian measure.

5. ____________ Given \( \cot x = \pi \), find the exact value of \( \sin(2x) \).

6. ____________ A triangle \( ABC \) has sides \( AC = \sqrt{3} \) and \( AB = 1 \). Angle \( C \) is 30°. Find the length of side \( BC \).

7. ____________ Find the exact value of \( \cos \left( \frac{7\pi}{12} \right) \).

8. ____________ Simplify \( \tan x (\csc x - \sin x) \).

9. ____________ Find the exact value of \( \sin \left( \sin^{-1} \left( -\frac{\pi}{4} \right) + \cos^{-1} \left( -\frac{\pi}{4} \right) \right) \).