Number Theory

Place all answers in the blank space provided. Calculators are permitted. You are not expected to answer all the questions.

1. Represent 1442016 by the use of the Fundamental Theorem of Arithmetic.

2. The symbol $\tau(n)$ denotes the number of positive divisors of $n$. Calculate $\tau(2016)$.

3. The symbol $\sigma(n)$ denotes the sum of positive divisors of $n$. Find $\sigma(2016)$.

4. The symbol $\varphi(n)$ denotes the number of positive integers not greater than $n$ that are relatively prime to $n$. Find $\varphi(2016)$.

5. Triangular numbers are defined as follows: $t_1 = 1$, $t_n = t_{n-1} + n$ for $n > 1$. Find $t_{2016}$.

6. A perfect number is a positive integer that is equal to the sum of its proper positive divisors. For example $6 = 1 + 2 + 3$, $28 = 1 + 2 + 4 + 7 + 14$. What is the next smallest perfect number?

7. Give an example of a positive integer $n$ such that $\varphi(n)$ is odd.

8. What are the largest twin primes both smaller than 1000?

9. Does the equation $a^{2016} - b^{2016} = c^{2016}$ have a solution in negative integers?

10. Does the following congruence $14x \equiv 4 \pmod{2016}$ have a solution?

11. The number $n = 4 \cdot 10^{2016} + 14$ is even, so divisible it is by 2. What is the next smallest prime divisor of $n$?

12. Find the remainder when $414^{2016}$ is divided by 7.

13. Express 2016 as a hexadecimal number.


15. Find a Pythagorean triple with 14 as one of the numbers.

16. The $\gcd(414, 2016) = 18$. Find a pair integers $(x, y)$ such that $414x + 2016y = 18$.

17. For which of the following prime numbers $p$, does the congruence $x^2 \equiv -1 \pmod{p}$ have a solution? Choices for $p$: 419, 2017, 5021, 9967.