Logic and Sets

Symbols

"\land" is the logical connective "and"
"\lor" is the logical connective "or"
"\neg p" is the negation of the statement p
"\implies" means implication
"\in" means "belongs to"
"\iff" means "if and only if"
"\equiv" means logical equivalence
"\forall" is the universal quantifier
"\exists" is the existential quantifier
"\mathbb{R}" means the set of real numbers

I. Logic. True or False or Unknown: (circle only one in each problem)

1. T F U If \( x \in A \) and \( x \in B \), then \( x \in (A \oplus B) \) (the symmetric difference of the two sets).

2. T F U \( 238^2 + 2016^2 = 2030^2 \implies (a + b)^2 = a^2 + b^2 \).

3. T F U If \( p \lor q \) is True, then \( p \implies q \) is ....

4. T F U If \( p \iff q \) is True, then what is the status of \( p \)?

5. T F U Suppose that \( p \) is False, \( q \) is True, and \( r \) is Unknown. What is the status of \( (p \land q) \lor (q \land r) \)?

6. T F U Suppose that \( p \) is False, \( q \) is Unknown, and \( r \) is False. What is the status of \( (p \iff q) \iff (q \land r) \)?

7. T F U The negation of \( (x \geq 2016 \iff f(x) \geq 36) \) is \( (x < 2016 \iff f(x) < 36) \).

8. T F U The converse of \( \neg p \iff q \) is \( q \iff \neg p \).

9. T F U A tautology is always false.

10. T F U The inverse of "If it is raining and sunny outside, then I am in Florida." is "If I am in Florida, then it is raining and sunny outside."

11. T F U \( (\forall y \in \mathbb{R})(\forall x \in \mathbb{R}) \left( (x > y) \implies (x + y \geq 0) \right) \).

12. T F U \( (\forall x \in \mathbb{R})(\exists y \in \mathbb{R}) \left( (x < y) \implies (x^2 < x + y - 1) \right) \).

13. T F U \( p \iff (q \land r) \equiv (p \iff q) \land (p \iff r) \).

14. T F U If we take the contrapositive of a contradiction, we get a tautology.

15. T F U There is exactly one False entry in the the column of the truth table for \( (p \land q) \lor r \).

16. T F U \( (p \implies q) \land r \equiv (p \land r) \implies (q \land r) \).
Symbols
\( \mathbb{N} = \{0, 1, 2, 3, \ldots\} \) is the set of natural numbers
\( U \) is the universal set
\( \emptyset \) is the empty set
\( \overline{S} \) is the complement of \( S \)
The set \( A - B \) is the set \( A \cap \overline{B} \)
\( \subseteq \) means 'is a subset of'
\( \subset \) means 'is a proper subset of'
n(\( S \)) denotes the number of elements in \( S \)
\( \in \) means 'is an element of'
\( A \times B \) denotes the Cartesian Product
\( \mathcal{P}(A) \) is the power set of a set \( A \)

II. Sets

Problems 1-3: Shade the appropriate area for each Venn Diagram.

1. \( (A \cap B) \cup (A \cap B) \)
2. \( A - (U - B) \)
3. \( (A \cap B) \cup (A \cap \overline{C}) \)

4. If there are 20 blue balls, 16 red ball, and 2016 black balls in a box together, how many balls do I have to pick in order to assure that I have at least five red balls and twenty black balls?

5. How many integers between 1 and 2016 are divisible by both 19 and 23?

For problems 6-13: Let \( A = \{1, 2, 8\}, B = \{5, 6, 7\}, C = \{0, 1, 2, 4, 8\}, D = \{1, 2, 5\}, E = \{0, 5\}, \) and \( U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\} \) be the universal set.

Using the sets just described, describe each of the following sets by listing its elements in \{ \}.

6. \( (B - \overline{A}) \cup C \)
7. \( (A - B) \cap ((A - C) \cup E) \)
8. \( (A \cup \overline{B}) \cap E \)
9. \( (A \cap E) \cap ((B \cup E) \cup (D \cap B)) \)

Using the sets described above, determine if the following statements are true or false.

10. \( n(C - A) = n(E) \)
11. \( n(C \times E) = 7 \)
12. \( n(\mathcal{P}(E)) = 4 \)
13. \( \emptyset \subseteq C \times E \)
14. (True/False) The set of transcendental numbers is countable.
15. (True/False) The empty set is an element of every power set.